Malaysian Journal of Science **21**: 149-158 (2002)

Analysis of first successful transmission in a buffered CSMA/CD

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ABSTRACT The buffered non-persistent CSMA/CD-type LAN proposed by Boyanov and Ruskov is a Markovian LAN model. The characteristics of the network were derived analytically by Lee and Pee. In this paper, we derive the distribution of the time required to get the first successful transmission for the model.

ABSTRAK Ethernet LAN jenis CSMA/CD tertimbal dan bukan tegar yang dicadangkan oleh Boyanov dan Ruskov merupakan model LAN berbentuk Markov. Hasil analisis tentang ciri-ciri rangkaian tersebut telah diterbitkan oleh Lee dan Pee. Dalam kertas ini kami menerbitkan taburan masa pemancaran berjaya yang pertama bagi model.

(CSMA/CD, local area networks, queueing systems, media access protocol)

INTRODUCTION

Ethernet LAN is a bus-based broadcast, multi-access communication network using carrier sense multiple access with collision detection protocol (CSMA/CD) [16]. Currently it has been standardised by the IEEE 802.3 committee [4], [5]. Various schemes of CSMA/CD schemes has been proposed to Ethernet LAN. There are the non-persistent CSMA/CD, the p-persistent CSMA/CD, and the 1-persistent CSMA/CD (which is a special case of the p-persistent CSMA/CD) [8]. In 1975, Tobagi and Kleinrock described various CSMA schemes (without collision detection) in the context of ground radio channels [15]. The model was then extended to the schemes of CSMA with collision detection mechanism [8]. Numerous related papers that had been written in the early years of 80s are [7] [9] [10] [14] [20]. However in the above papers, the CSMA/CD protocol has been analysed under the assumption that each user has a buffer capacity of one unit. In practice, this is a rather tight limitation [23].

In 1985, Takagi and Kleinrock derived explicitly the mean packet delay (including the queueing and retransmission delays) for the two users of buffered slotted p-persistent CSMA/CD model [32].

Tasaka provided an approximate performance analysis of a slotted non-persistent CSMA/CD system with a finite number of users, each having a buffer of finite capacity [18]. The model was analysed using an approximate analytical technique called equilibrium point analysis (EPA) which is a diffusion approximation [1], [19]. The throughput and average packet delay characteristics were obtained and system stability behaviour was demonstrated.

Apostolopoulos and Protonstarios in [23] described a two-dimensional Markovian analysis of a slotted DFT (delay-first-transmission) CSMA/CD with finite buffers. This is an extension of the related work for DFT slotted multiple access protocols with finite buffers in Sykas, Karvelas and Protonotarios, [6]. The queue length of the tagged station and the number of the other active stations were chosen as the elements of the state vector. It is noted that, the approach in [6] and [23] required the iterative scheme to obtain performance measures. Ganz and Chlamtac proposed a simpler approach to slot multiple access protocols with finite buffer [2]. Their approach required only the solution of linear simultaneous equations. A setback of these approaches ([2], [6], [23]) is that it cannot be applied to a system with infinite buffers.

Takine, Takahashi and Hasegawa [21], using the idea of Apostolopoulos and Protonotarios in [22], decomposed the CSMA/CD by treating each station as an independent M/G/1 queueing system. In their system it was assumed that there were finite numbers of homogenous stations, each having infinite buffers. Mean message delay and stability of system were discussed in this paper.

Another buffered CSMA/CD model was proposed by Boyanov, Ruskov, Yanev and Dimitrov [13]. This paper describes the characteristics of non-persistent CSMA/CD with the assumption of infinite number of stations each having infinite buffer capability. They modelled the non-persistent CSMA/CD as a Markov process and described network characteristics by considering the voice and data requests. Important characteristics of the network have been obtained. A simplified version of the model considering only the data request was discussed in [11]. One of the basic characteristic of the model in [11] was studied in [3]. The characteristics of the network in [11] were also derived analytically using the generating function approach in [17].

The objective of this paper is to obtain the probability density function (p.d.f.) $T(t)$ of the time t required to get the first successful transmission. Let $t = 0$ be a time when the system is in a stationary state. Let $T_{in}(t)$ be the p.d.f. of the time t required to get the first successful transmission given that at time $t = 0$, the system is in state (i,n), that is the channel is in state i and the number of repeated requests is n. Furthermore let $P_{i\,n}$ ([3] [11] [17]) denote the probability that at $t = 0$ the system is in state (i,n). Then

$$T(t) = \sum_{i=0}^{2}\sum_{n=0}^{\infty} P_{i\,n} T_{i\,n}(t) \qquad (1.1)$$

In section 2 and 3 we show how $T_{i\,n}(t)$ can be derived. The numerical results and conclusions for $T(t)$ are presented in section 4.

Derivation of Difference Equations for $T_{i\,n}(t)$

Let $\varphi(t) = \lambda e^{-\lambda t}$ be the p.d.f. of the inter-arrival time for the primary requests, $\phi_n(t) = \xi n e^{-\xi n t}$ the p.d.f. of the inter-arrival time for repeated requests, in which n is the number of repeated requests, $\alpha(t) = \omega e^{-\omega t}$ the p.d.f. of the attempt time to occupy the channel (collision window), $\gamma(t) = \mu e^{-\mu t}$ the p.d.f. of the time for transmitting the data packet.

Furthermore let

$$f_B(t) = \int_t^{\infty} f(x)dx, \quad f_F(t) = \int_t^{\infty} f(x)dx \Big/ \mu_f,$$

(μ_f is the mean of p.d.f. $f(t)$ and $f_{E\alpha}(t) = f_{E\alpha}(t) f(t) e^{-at}$.

A difference equation for $T_{0\,n}(t)$ may be derived as follows. First let us see how we may get a first successful transmission at time t if the system is initially in the state $(0,n)$ at $t = 0$. We note that after $t = 0$, the first request may be a primary request, or a repeated request if $n = 1$. If the first request is a primary request, then the attempt to occupy the channel may be successful (see Figure 1 (a)), or not successful due to the arrival of another primary request (see Figure 1 (b)) or the arrival of a repeated request (see Figure 1 (c)). If the first request is a repeated request, then the attempt to occupy the channel may be successful (see Figure 1 (d)) or not successful due to the arrival of a primary request (see Figure 1 (e)) or the arrival of another repeated request (see Figure 1 (f)) if $n \geq 2$. Once the attempt is successful, a successful transmission will occur at time t. But whenever an attempt to occupy the channel is interrupted, the channel will go to state 0 and the number of accumulated repeated requests will be updated accordingly (to n' say). Starting from the state $(0,n')$ at the time of collision (t_2 say), transmission should then occur at $t - t_2$. The p.d.f. of $t - t_2$ is given by $T_{0\,n'}(t - t_2)$.

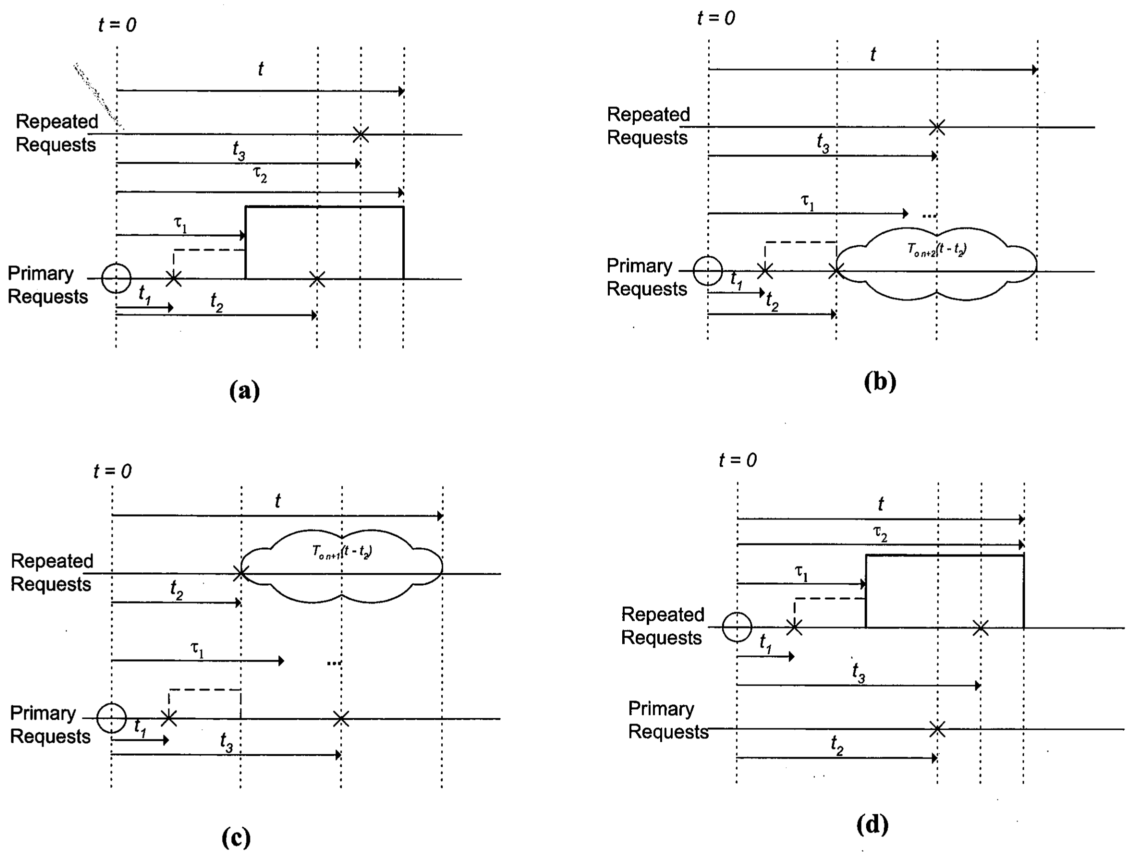
From Figure 1, we see that

$$T_{0\,n}(t) = \int_0^t \varphi_F(t_1)dt_1 \int_{t_1}^t \alpha(\tau_1 - t_1)d\tau_1 \gamma(t - \tau_1) \int_{\tau_1}^{\infty} \varphi(t_2 - t_1)dt_2 \int_{\tau_1}^{\infty} \phi_n(t_3)dt_3$$

$$
\begin{aligned}
&+\int_0^t \varphi_F(t_1)dt_1 \int_{t_1}^{\infty} \alpha(\tau_1 - t_1)d\tau_1 \int_{t_1}^{\min(\tau_1,t)} \varphi(t_2 - t_1)dt_2 \int_{t_2}^{\infty} \phi_{nF}(t_3)dt_3 T_{0\,n+2}(t-t_2) \\
&+\int_0^t \varphi_F(t_1)dt_1 \int_{t_1}^{\infty} \alpha(\tau_1 - t_1)d\tau_1 \int_{t_1}^{\min(\tau_1,t)} \phi_{nF}(t_2)dt_2 \int_{t_2}^{\infty} \varphi(t_3 - t_1)dt_3 T_{0\,n+1}(t-t_2) \\
&+\int_0^t \phi_{nF}(t_1)dt_1 \int_{t_1}^{t} \alpha(\tau_1 - t_1)d\tau_1 \gamma(t-\tau_1) \int_{\tau_1}^{\infty} \varphi_F(t_2)dt_2 \int_{\tau_1}^{\infty} \phi_{n-1}(t_3 - t_1)dt_3 \\
&+\int_0^t \phi_{nF}(t_1)dt_1 \int_{t_1}^{\infty} \alpha(\tau_1 - t_1)d\tau_1 \int_{t_1}^{\min(\tau_1,t)} \varphi_F(t_2)dt_2 \int_{t_2}^{\infty} \phi_{n-1}(t_3 - t_1)dt_3 T_{0\,n+1}(t-t_2) \\
&+\int_0^t \phi_{nF}(t_1)dt_1 \int_{t_1}^{\infty} \alpha(\tau_1 - t_1)d\tau_1 \int_{t_1}^{\min(\tau_1,t)} \phi_{n-1}(t_2 - t_1)dt_2 \int_{t_2}^{\infty} \varphi_F(t_3)dt_3 T_{0\,n}(t-t_2)\,,\ n = 0, 1, \ldots \qquad (1)
\end{aligned}
$$

which can be simplified to

$$
\begin{aligned}
T_{0\,n}(t) = {} & \varphi_{E_{n\xi}} * \alpha_{E_{\lambda+n\xi}} * \gamma(t) + \lambda\varphi_{E_{n\xi}} * (\alpha_B)_{E_{\lambda+n\xi}} * T_{0\,n+2}(t) \\
& + \xi n \varphi_{E_{n\xi}} * (\alpha_B)_{E_{\lambda+n\xi}} * T_{0\,n+1}(t) + \phi_{nE_\lambda} * \alpha_{E_{\lambda+(n-1)\xi}} * \gamma(t) \\
& + \lambda\phi_{nE_\lambda} * (\alpha_B)_{E_{\lambda+(n-1)\xi}} * T_{0\,n+1}(t) + \xi(n-1)\phi_{nE_\lambda} * (\alpha_B)_{E_{\lambda+(n-1)\xi}} * T_{0\,n}(t) \quad . n = 0, 1, \ldots \qquad (2)
\end{aligned}
$$



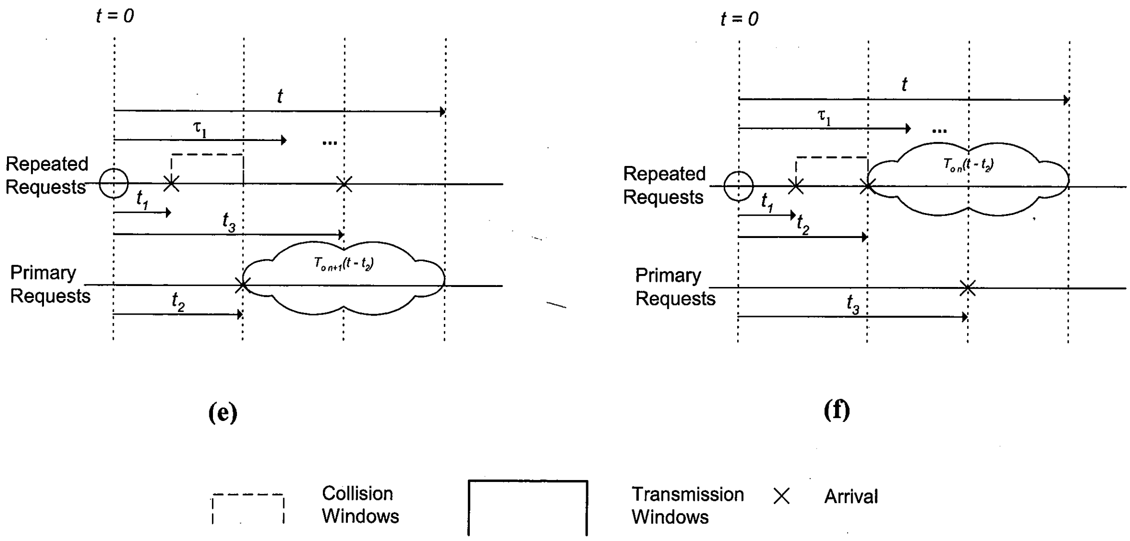


Figure 1. Diagrams relevant for the derivation of $T_{0n}(t)$.

We next derive a difference equation for $T_{1n}(t)$.

We note that the attempt which has not ended at $t = 0$ may eventually be successful (see Figure 2 (a)), or interrupted by a primary request (see Figure 2 (b)), or a repeated requested if $n \geq 1$ (see Figure 2 (c))

A successful attempt to occupy the channel will be followed by a successful transmission at time t. On the contrary, a collision (at time t_1 say) will bring the system to the state $(0, n')$ where n' is the updated number of repeated requests at time t_1. Starting from the state $(0, n')$ at time t_1, the first successful transmission will occur at time t. The difference $t - t_1$ has a p.d.f. given by $T_{1n'}(t - t_1)$.

From Figure 2, we see that

$$T_{1n}(t) = \int_0^t \alpha_F(\tau_1)d\tau_1 \gamma(t-\tau_1)\int_{\tau_1}^{\infty}\varphi_F(t_1)dt_1\int_{\tau_1}^{\infty}\phi_{nF}(t_2)dt_2$$
$$+\int_0^{\infty}\alpha_F(\tau_1)d\tau_1\int_0^{\min(t,\tau_1)}\varphi_F(t_1)dt_1\int_{t_1}^{\infty}\phi_{nF}(t_2)dt_2 T_{0\,n+2}(t-t_1)+$$
$$\int_0^{\infty}\alpha_F(\tau_1)d\tau_1\int_0^{\min(t,\tau_1)}\phi_{nF}(t_1)dt_1\int_{t_1}^{\infty}\varphi_F(t_2)dt_2 T_{0\,n+1}(t-t_1),$$
$$n = 0, 1, \ldots \qquad (3)$$

which can be simplified to

$$T_{1n}(t) = (\alpha_F)E_{\lambda\eta\xi} * \gamma(t) + \lambda((\alpha_F)B)\,E_{\lambda\eta\xi} * T_{0n+2}(t) + \xi\eta\,((\alpha_F)B)\,E_{\lambda\eta\xi} * T_{0n+1}(t).$$
$$n = 0, 1, \ldots \qquad (4)$$

$T_{2n}(t)$ is simply given by $T_{2n}(t) = \mu e^{-\mu t}$.
$$n = 0, 1, \ldots \qquad (5)$$

3. Solution of the Difference Equations $T_{in}(t)$

On taking Laplace transforms of both sides of equation (2) we get

$$\hat{T}_{0n}(s) = \frac{\lambda\mu\omega}{(s+\xi n+\lambda)(s+\lambda+\xi n+\omega)(s+\mu)}$$
$$+\frac{\lambda^2\hat{T}_{0\,n+2}(s)}{(s+\lambda+\xi n)(s+\lambda+\xi n+\omega)}+\frac{\lambda\xi n\hat{T}_{0\,n+1}(s)}{(s+\lambda+\xi n)(s+\lambda+\xi n+\omega)}$$
$$+\frac{\xi n\mu\omega}{(s+\lambda+\xi n)(s+\lambda+\xi(n-1)+\omega)(s+\mu)}$$
$$+\frac{\lambda\xi n\hat{T}_{0\,n+1}(s)}{(s+\lambda+\xi n)(s+\lambda+\xi(n-1)+\omega)}$$
$$+\frac{\xi^2 n(n-1)\hat{T}_{0n}(s)}{(s+\lambda+\xi n)(s+\lambda+\xi(n-1)+\omega)}.$$
$$n = 0, 1, \ldots \qquad (6)$$

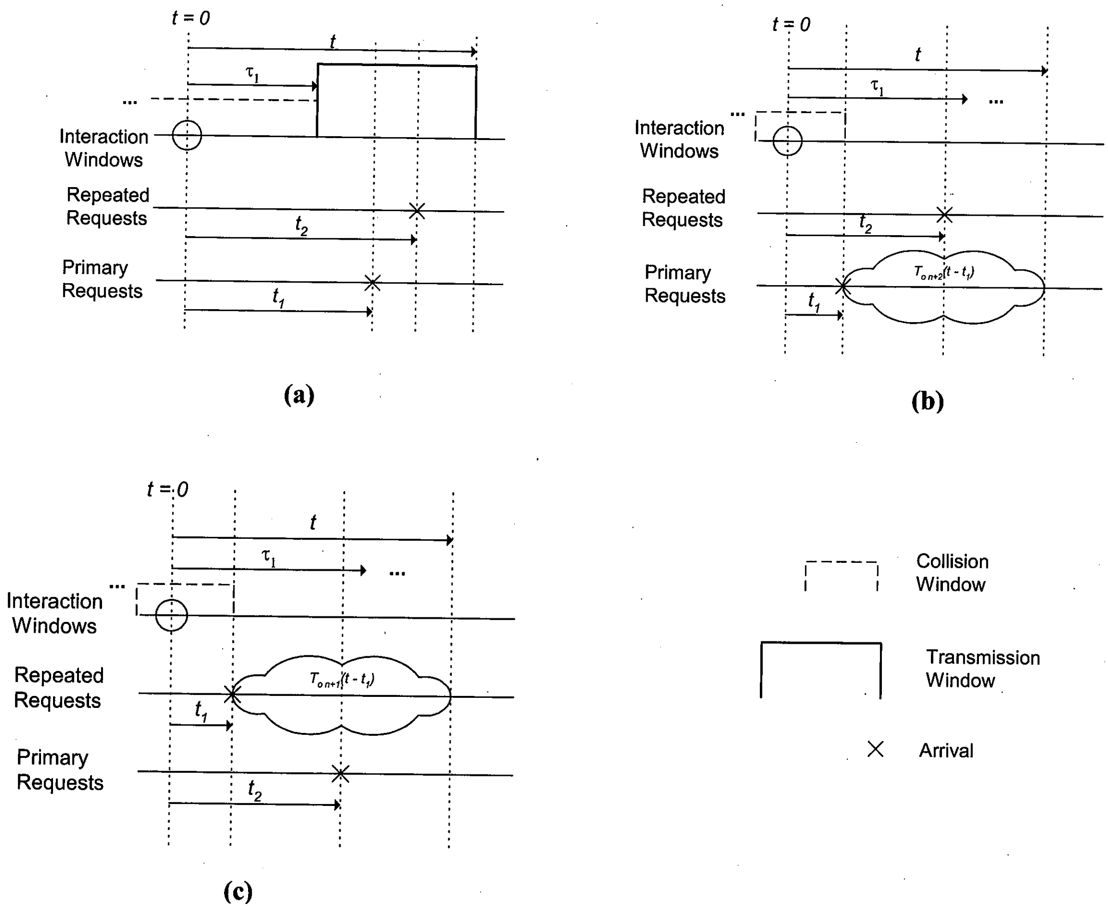


Figure 2. Diagrams relevant for the derivation of $T_{1n}(t)$.

Let $n = k$ be a value which is large enough such that $\xi k >> \lambda$. Then among the six expressions on the right side of (6), the dominating expressions will be the fourth and the last expressions. Therefore $\hat{T}_{0k}(s)$ can be approximated by

$$\hat{T}_{0k}(s) = \frac{\xi k\mu\omega}{(s+\mu)[(s+\xi k)(s+\xi(k-1)+\omega)-\xi^2 k(k-1)]}. \quad (7)$$

We next invert $\hat{T}_{0k}(s)$. First let

$$B = (s+\lambda+\xi(k+1)+\omega+\mu). \quad (8)$$

Then $\hat{T}_{0k}(s)$ in (7) can be rewritten as

$$\hat{T}_{0k}(s)(1+\frac{a'_{k1}}{B}+\frac{a'_{k2}}{B^2}+\frac{a'_{k3}}{B^3}) = \xi k\omega\mu/B^3, \quad (9)$$

where

$\alpha^1_{k1} = -(3\lambda + 2\omega + 2\mu + \xi(k+4))]$, $\alpha^1_{k2} = (\lambda + \omega + \mu + \xi)(\lambda + \mu + 2\xi) - \xi^2 k(k-1) + (\lambda + \omega + \xi(k+1))[(\lambda+\mu + \omega + \xi) + (\lambda + \mu + 2\xi)]$, $\alpha^1_{k3} = -(\lambda + \omega + \xi(k+1))[(\lambda + \omega + \mu + \xi) \times (\lambda + \mu + 2\xi) - \xi^2 k(k-1)]$.

Let

$$\hat{T}_{0k}(s) = \sum_{i=0}^{\infty}\frac{c_{ki}}{B^i}. \quad (10)$$

Then from (9) and (10) we get

$$\sum_{i=0}^{\infty}\frac{c_{ki}+a'_{k1}c_{ki-1}+a'_{k2}c_{ki-2}+a'_{k3}c_{ki-3}}{B^i} = \frac{\xi k\mu\omega}{B^3}, \quad (11)$$

where $c_{k\,i} = 0$ for $i < 0$.

By comparing the coefficients of B^{-i}, $i = 0, 1, \ldots$ on both sides of (11), we get

$$c_{k\,0} = c_{k\,1} = c_{k\,2} = 0, \tag{12}$$

$$c_{k\,3} = \xi k\mu\omega, \tag{13}$$

$$c_{k\,i} = -a'_{k\,1}c_{k\,i-1} - a'_{k\,2}c_{k\,i-2} - a'_{k3}c_{k\,i-3} \quad i = 4, 5, \ldots \tag{14}$$

By inverting the Laplace transform in (10), we get

$$T_{0\,k}(t) = \sum_{i=3}^{\infty}\frac{c_{k\,i}t^{i-1}}{(i-1)!}e^{-(\lambda+\mu+\omega+\xi(k+1))t}. \tag{15}$$

When $n \geq k+1$, it is obvious that $\xi k >> \lambda$ implies that $\xi n >> \lambda$. Therefore the series expansion for $T_{0\,n}(t)$, $n \geq k+1$, can be obtained in a similar way as that for $T_{0\,k}(t)$.

For $n < k$, $T_{0\,n}(t)$ can be derived from $T_{0\,k}(t)$ and $T_{0\,k+1}(t)$ as follows. By using $B = (s + \lambda + \xi(k+1) + \omega + \mu)$ as defined in (8), we can rewrite (6) as

$$\hat{T}_{0\,n}(s)\left[1 + \frac{a'_{n\,1}}{B} + \frac{a'_{n\,2}}{B^2} + \frac{a'_{n\,3}}{B^3} + \frac{a'_{n\,4}}{B^4}\right]$$
$$= \frac{b'_{n\,1}}{B^3} + \frac{b'_{n\,2}}{B^4} + \left[\frac{c'_{n\,1}}{B^2} + \frac{c'_{n\,2}}{B^3} + \frac{c'_{n\,3}}{B^4}\right]\hat{T}_{0\,n+2}(s)$$
$$+ \left[\frac{d'_{n\,1}}{B^2} + \frac{d'_{n\,2}}{B^3} + \frac{d'_{n\,3}}{B^4}\right]\hat{T}_{0\,n+1}(s), \quad n < k \tag{16}$$

where

$$a'_{n\,1} = -(y_n + y_{n-1} + x_{k+1} + w_n),$$
$$a'_{n\,2} = y_n x_{k+1} + (w_n + y_{n-1})(y_n + x_{k+1}) + w_n y_{n-1} - \xi^2 n(n-1),$$
$$a'_{n\,3} = -[y_n x_{k+1}(w_n + y_{n-1}) + (w_n y_{n-1} - \xi^2 n(n-1))(y_n + x_{k+1})],$$
$$a'_{n\,4} = y_n x_{k+1}(w_n y_{n-1} - \xi^2 n(n-1)),$$
$$b'_{n\,1} = \xi n\omega\mu + \lambda\omega\mu,$$
$$b'_{n\,2} = -(\xi n\omega\mu y_n + \lambda\omega\mu y_{n-1}),\ c'_{n\,1} = \lambda^2,$$
$$c'_{n\,2} = -\lambda^2(x_{k+1} + y_{n-1}),$$
$$c'_{n\,3} = \lambda^2 x_{k+1}y_{n-1},\ d'_{n\,1} = 2\lambda\xi n,$$
$$d'_{n\,2} = -\lambda\xi n(2x_{k+1} + y_{n-1} + y_n),$$
$$d'_{n\,3} = \lambda\xi n x_{k+1}(y_{n-1} + y_n).$$

$$w_n = \omega + \mu + \xi(k + 1 - n),$$
$$x_n = \lambda + \omega + \xi n,\ y_n = \mu + \xi(k + 1 - n).$$

Let

$$\hat{T}_{0\,n}(s) = \sum_{i=0}^{\infty}\frac{c_{n\,i}}{B^i},\ n = 0, 1, \ldots.$$

By using the series expansions for $\hat{T}_{0\,k}(s)$ and $\hat{T}_{0\,k+1}(s)$, we can find the following series expansion for $T_{0\,n}(t)$

$$T_{0\,n}(t) = \sum_{i=3}^{\infty}\frac{c_{n\,i}t^{i-1}}{(i-1)!}e^{-(\lambda+\mu+\omega+\xi(k+1))t}, \quad n < k \tag{17}$$

where

$$c_{n\,0} = c_{n\,1} = c_{n\,2} = 0, \tag{18}$$

$$c_{n\,3} = b'_{n\,1}, \tag{19}$$

$$c_{n\,4} = b'_{n\,2} - a'_{n\,1}c_{n\,3}, \tag{20}$$

$$c_{n\,i} = -a'_{n1}c_{n\,i-1} - a'_{n2}c_{n\,i-2} - a'_{n3}c_{n\,i-3} - a'_{n4}c_{n\,i-4}$$
$$+ c'_{n\,1}c_{n+2\,i-2} + c'_{n\,2}c_{n+2\,i-3} + c'_{n\,3}c_{n+2\,i-4}$$
$$+ d'_{n\,1}c_{n+1\,i-2} + d'_{n\,2}c_{n+1\,i-3} + d'_{n\,3}c_{n+1\,i-4}. \quad i = 5, 6, \ldots \tag{21}$$

To find the $T_{1\,n}(t)$, we first take Laplace transforms on both sides of (4) to get

$$\hat{T}_{1\,n}(s) = \frac{\mu\omega}{(s + \lambda + \xi n + \omega)(s + \mu)} + \frac{\lambda\hat{T}_{0\,n+2}(s)}{(s + \lambda + \xi n + \omega)} + \frac{\xi n\hat{T}_{0\,n+1}(s)}{(s + \lambda + \xi\,n + \omega)}. \quad n = 0, 1, \ldots \tag{22}$$

This can be rewritten as

$$\hat{T}_{1n}(s)\left(1+\frac{a''_{n1}}{B}+\frac{a''_{n2}}{B^2}\right)=\frac{b''_{n1}}{B^2}$$
$$+\left(\frac{c''_{n1}}{B}+\frac{c''_{n2}}{B^2}\right)\hat{T}_{0n+2}(s)$$
$$+\left(\frac{d''_{n1}}{B}+\frac{d''_{n2}}{B^2}\right)\hat{T}_{0n+1}(s).$$
$$n=0,1,... \quad (23)$$

where

$a''_{n1}=-[(\mu+\xi(k+1-n)+(\lambda+\omega+\xi(k+1)]$,

$a''_{n2}=(\mu+\xi(k+1-n))(\lambda+\omega+\xi(k+1))$, $b''_{n1}=\omega\mu$,

$c''_{n1}=\lambda$, $c''_{n2}=-\lambda(\omega+\lambda+(k+1)\xi)$, $d''_{n1}=\xi n$,

$d''_{n2}=-\xi n(\omega+\lambda+\xi(k+1))$.

Let,

$$\hat{T}_{1n}(s)=\sum_{i=0}^{\infty}\frac{d_{ni}}{B^i},\ n=0,1,\ldots .$$

By using the same technique as indicated above, we get

$$T_{1n}(t)=\sum_{i=2}^{\infty}\frac{d_{ni}t^{i-1}}{(i-1)!}e^{-(\lambda+\mu+\omega+\xi(k+1))t}$$
$$n=0,1,... \quad (24)$$

where

$$d_{n0}=d_{n1}=0, \quad (25)$$

$$d_{n2}=b''_{n1}, \quad (26)$$

$$d_{ni}=-a''_{n1}d_{ni-1}-a''_{n2}d_{ni-2}+c''_{n1}c_{n+2\,i-1}$$
$$+c''_{n2}c_{n+2\,i-2}+d''_{n1}c_{n+1\,i-1}+d''_{n2}c_{n+1\,i-2}.$$
$$i=3,4,... \quad (27)$$

NUMERICAL RESULTS AND CONCLUSIONS

Consider the case when the transmission intensity is $\mu=2500(packet/s)$, the retransmission intensity $\xi=4000(packet/s)$, the intensity of collision window $\omega=2000(packet/s)$ and the arrival intensity of primary requests λ=100, 250, 500, 650, and *700(packet/s)*.

Figure 3 shows the curves of $T(t)$, i.e. the distribution of the time required for the occurrence of the first successful transmission. If λ is small (for example $\lambda=100$) then collision will not be frequent, and $T(t)$ may be approximated by the following $\tilde{T}(t)$ obtained by assuming that there is no collision

$$\tilde{T}(t)\approx P_0[\lambda e^{-\lambda t}*\omega e^{-(\omega+\lambda)t}*\mu e^{-\mu t}]$$
$$+P_1[\omega e^{-(\omega+\lambda)t}*\mu e^{-\mu t}]+P_2\mu e^{-\mu t}. \quad (28)$$

When $\lambda=100$, $T(t)$ and $\tilde{T}(t)$ are found to be close to each other (see Figure 4).

If λ is large, then the collision will tend to occur, causing failure to the attempt to occupy the channel. But on the other hand, collision will result in more repeated requests which in turn increase the overall intensity of the arrival stream. With more frequent attempts from the primary and repeated requests, the likelihood of successfully occupying the channel within a given period is increased. Figure 3 shows that the combined effect of large value of λ (for example λ = 650, 700) is that the first successful transmission will tend to occur earlier.

Figure 5 (a) shows the curves for $T_{0n}(t)$ when λ =*500*, μ = 2500, ξ = 4000 and ω = 2000. If at $t=0$, the channel is in state *0* and there is no repeated request, then each of the first and second requests will be a primary request. Now as λ =*100* is relatively small compared to ω = 2000, it is quite unlikely that the first and second primary requests will collide. This means we may approximate $T_{00}(t)$ by the following $\tilde{T}_{00}(t)$, which is obtained by assuming that the first primary request will be able to occupy the channel successfully and get it self transmitted

$$\tilde{T}_{00}(t)\approx\lambda e^{-\lambda t}*\omega e^{-(\omega+\lambda)t}*\mu e^{-\mu t}. \quad (29)$$

The function $T_{00}(t)$ is found to be very close to $\tilde{T}_{00}(t)$ when λ =*100* (see Figure 6). Deviation of $\tilde{T}_{00}(t)$ from $T_{00}(t)$ becomes obvious when λ =*500*.

The curves for $T_{00}(t)$ and $T_{01}(t)$ shown in Figures 5 (a) are very much different. This is because with only one extra repeated request, the combined intensity of the input stream is increased tremendously from $\lambda = 500$ to $\lambda + \xi = 4500$.

In Figures 5 (a) and 5 (b), the curves of $T_{0n}(t)$ and $T_{1n}(t)$ when $n \geq 10$ are very much alike with a mean value of about *1.39E-03* (for the case when λ=*500*). A possible explanation is as follows. When $n \geq 10$, $n\xi$ will be at least *40000* which is *80* times the value of λ=*500*. This means it is sufficiently large such that the primary requests may be neglected and we may consider the input as being made up of only repeated requests.

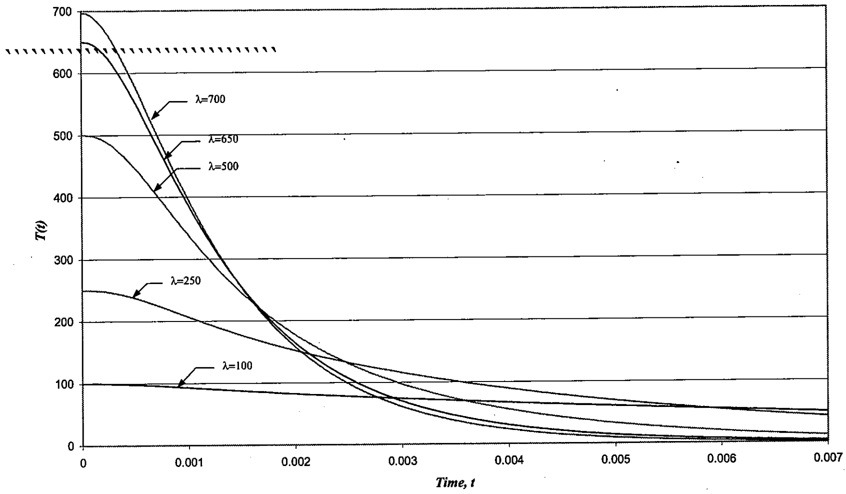


Figure 3. P.d.f. *T(t)* of the time of first successful transmission for the case when λ =100, 250, 500, 650 and 700, μ =2500, ξ = 4000 and ω =2000.

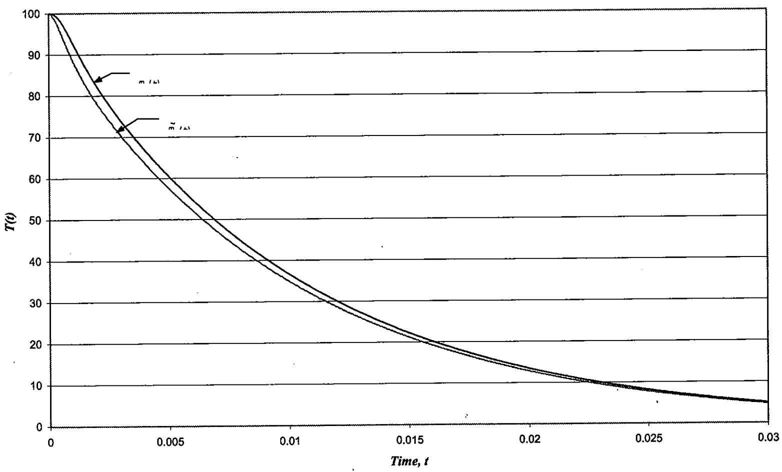


Figure 4. *T(t)* and $\widetilde{T}(t)$, when λ =*100*, μ =*2500*, ξ = *4000 and* ω =*2000.*

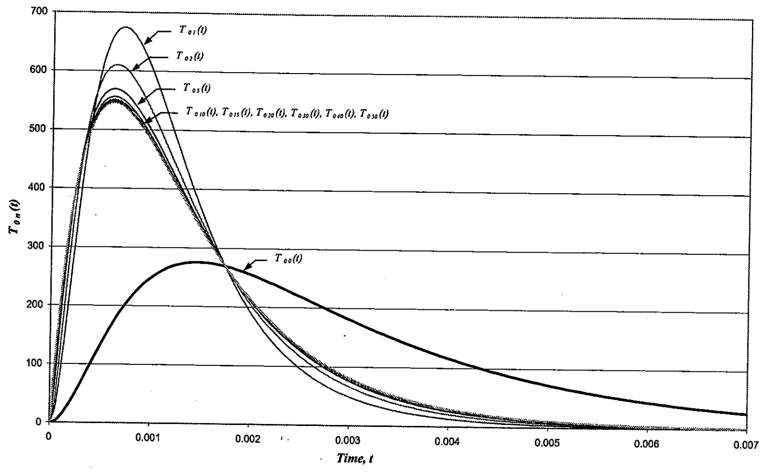


Figure 5 (a). P.d.f. $T_{0\,n}(t)$, when λ = *500*,μ *=2500*,ξ *= 4000 and* ω *=2000.*

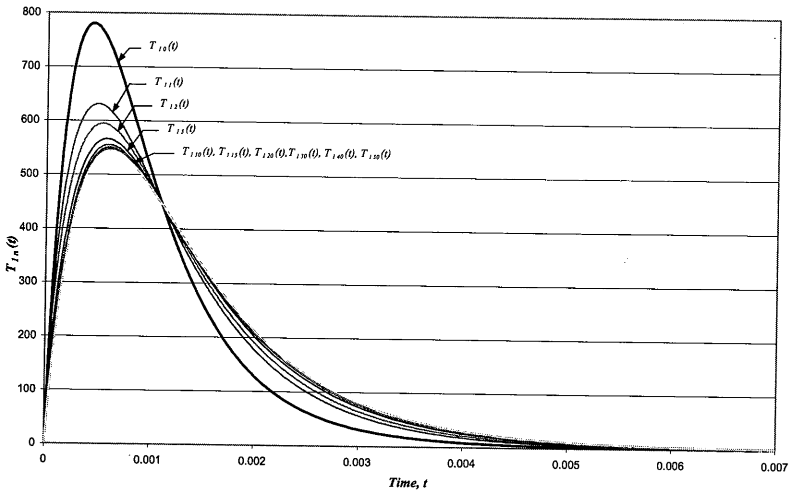


Figure 5 (b). P.d.f. of $T_{1\,n}(t)$, when λ = *500*,μ *=2500*,ξ *= 4000 and* ω *=2000.*

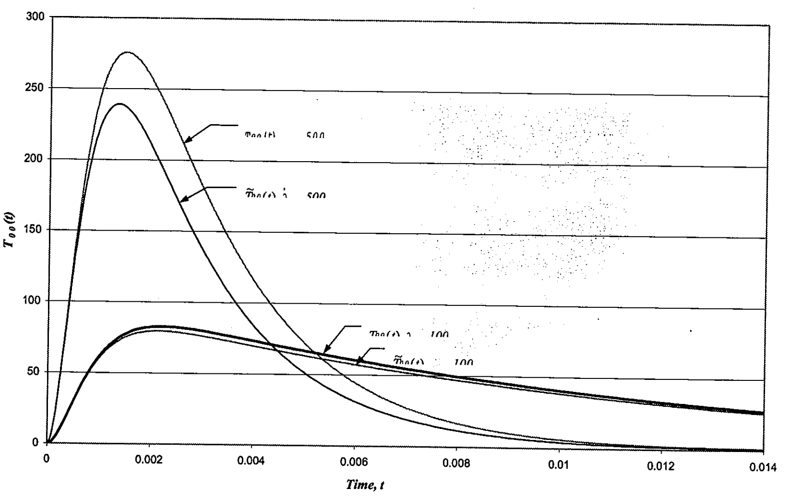


Figure 6. $T_{0\,0}(t)$ and $\widetilde{T}_{0\,0}(t)$, when λ *=100 and 500*,μ *=2500*,ξ *= 4000 and* ω *=2000.*

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